

Lab 2g: Dynamic Response of a Model Vehicle Suspension

OBJECTIVE

You are to measure and study the dynamic characteristics of a model vehicle suspension system treating it as a spring-mass-damper system with a single degree of freedom. You will use an experimental model representing one quarter of a car suspension. More thorough analysis of a full suspension system such as those shown in Figure 1 are quite complex involving all four tire/suspension systems acting independently. The quarter-car suspension model can be

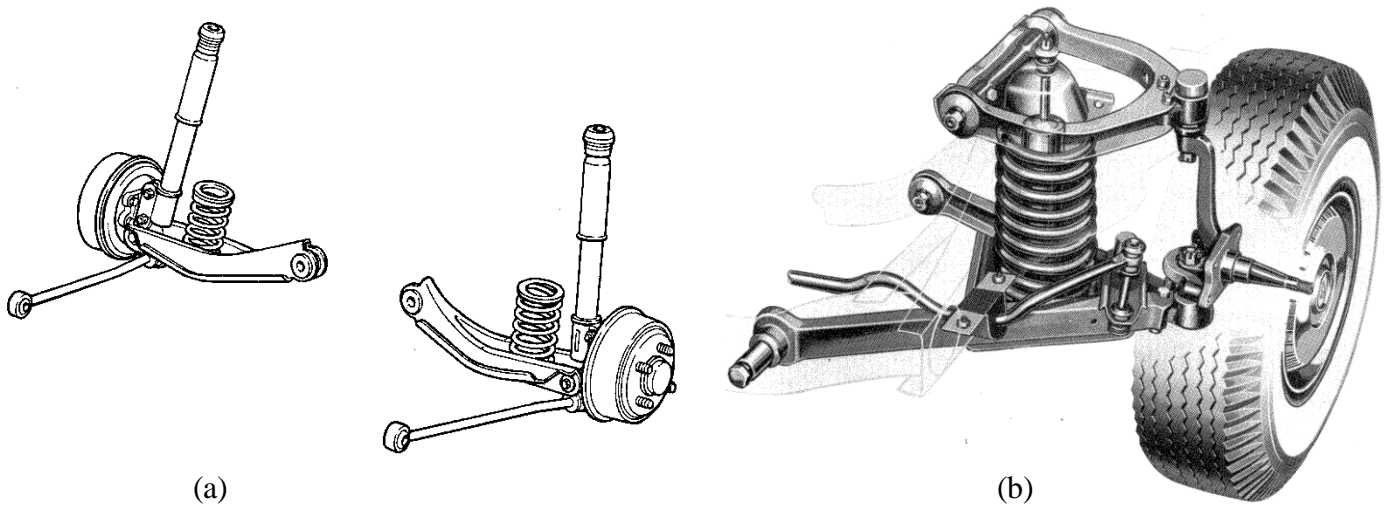


Figure 1. Examples of suspension systems (a) MacPherson strut rear suspension and (b) A-arm front suspension from Ford vehicles (from reference [2]).

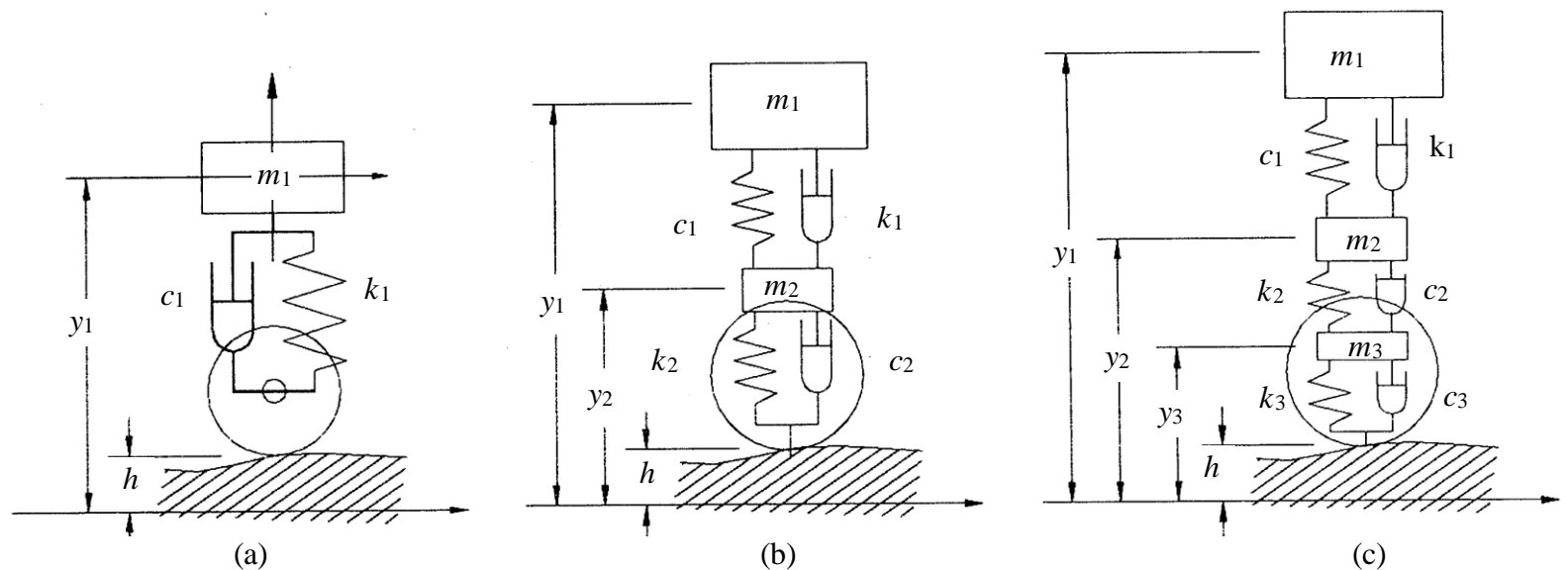


Figure 2. Modeling one quarter of a vehicle suspension system as second order system using (a) one, (b) two and (c) three degrees of freedom (from reference [3]).

considered in the three levels of complexity shown in Figure 2. The one-degree of freedom model shown in Figure 2(a) considers displacement y_1 of the sprung mass m_1 of the vehicle and the primary suspension stiffness k_1 and damping c_1 only. Here the unsprung mass (mass of the wheels and other components such as lower control arms) and the mass of the tires are not considered. The two degree of freedom model shown in Figure 2(b) accounts for the dynamics of the unsprung mass as well and introduces a second equation of motion and degree of freedom for the displacement y_2 of the unsprung mass m_2 . In this model, the tires are massless springs and dampers with k_2 and c_2 . A three-degree of freedom model is shown in Figure 2(c) where the dynamics of the tires are added to the analysis by treating them as a mass spring damper as well. Fortunately for you, this lab examines the suspension system using the single degree of freedom model shown Figure 2(a) and hence the subscript “1” used in that figure will be dropped. On the first day of lab you will perform calibration and, time permitting, can begin measurements to determine the characteristics of the system: the effective inertia of the system control arm, the spring stiffness k and damping coefficient c . Then on the second day of lab you will complete the measurements to determine the characteristics and measure the system’s driven response.

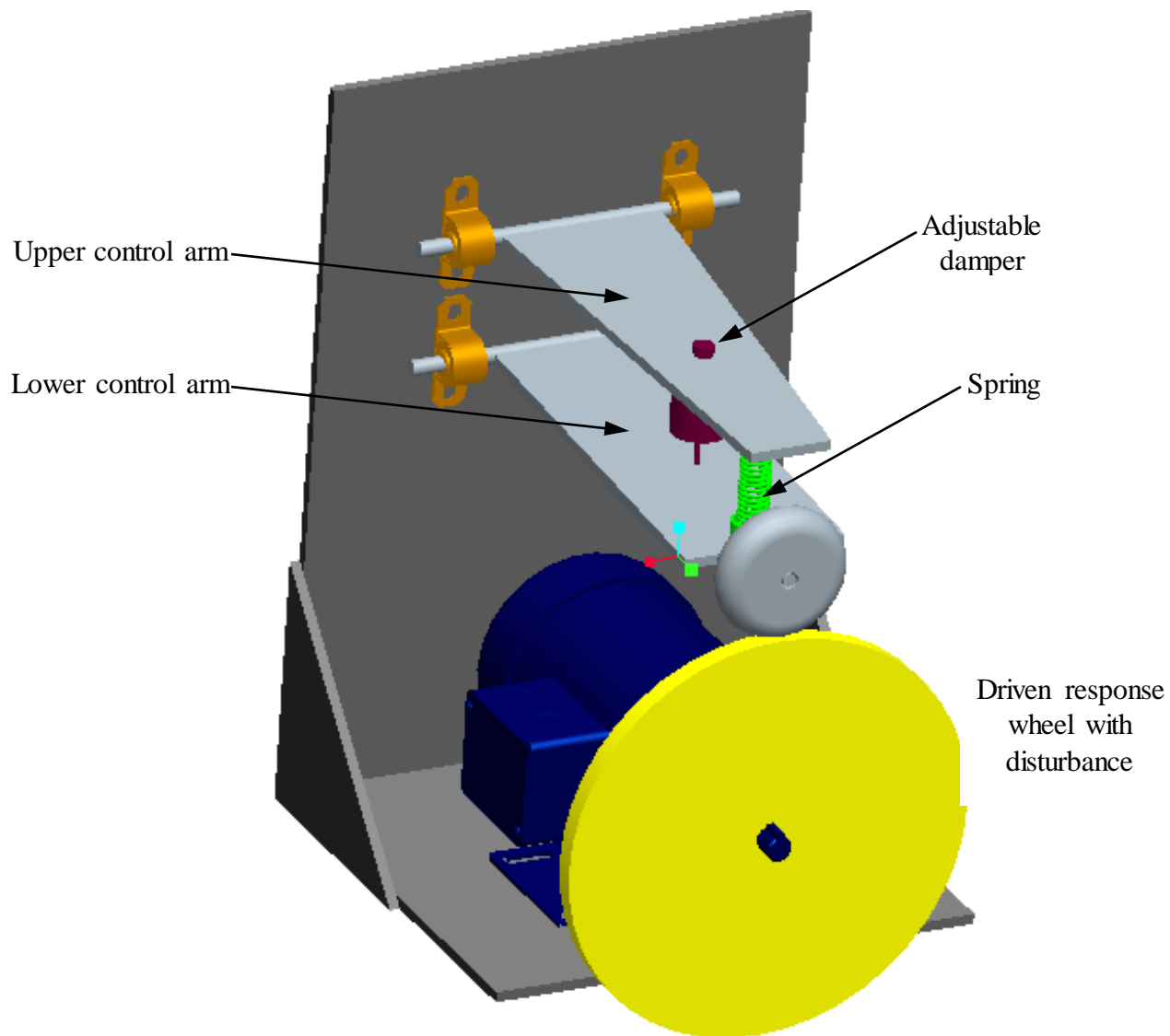
EQUIPMENT

Figure 3. Experimental test stand for one quarter of a vehicle suspension system.

The suspension experimental test stand is shown in Figure 3. The stand uses an upper and lower control arm with a spring and adjustable damper set between them. The upper control arm has a threaded rod (not shown in Figure 3) to hold the sprung mass applied to the system. Masses should always be screwed down using the wing nut for safety and to prevent noisy signals.

The following equipment will also be used:

- Two AK9-10 proximity probes to sense the position/displacement of the lower control arm.
- An HP/Agilent 3620A power supply to supply sensor excitation voltage to the proximity probes.
- Two Vernier 25G accelerometers mounted to the upper and lower control arms.
- A computer with a data acquisition system using an NI-6009 DAQ and a Vernier DAQ.

PROCEDURE FOR DAY 1

- a. Log into the computer and download the LabView VI from the course website. This program will be used to record measurements from the proximity sensors and accelerometers.
- b. Calibrate the proximity sensor (see the section “CALIBRATION OF PROXIMITY SENSOR” that follows for this procedure). Record the readings using the LabView VI and generate a calibration plot in Excel. The linear curve from this plot will be used to convert voltage to displacement later in the lab.
- c. Calibrate the accelerometers by determining the zero-g voltage. Expose the accelerometers to the Earth’s gravitational field (i.e. do not apply any motion to the accelerometer). Record the mean voltage output on the VI. Note that the accelerometers must be on a level surface for accurate calibration. The accelerometer voltage output converts directly to voltage (9.8 m/s^2 will read as approximately 9.8V). Any difference in the measured voltage represents a systematic error. This will be the only calibration point for the Vernier accelerometers. Therefore, only this systematic error will be considered for uncertainty analysis.

REFERENCES

1. Figliola, R. S. and Beasley, D. E., *Theory and Design for Mechanical Measurements*, 4th Ed., Wiley, 2006, potentiometer on pp. 199-201 and 474-477, and accelerometers pp. 482-489.
2. Gillespie, T. D., *Fundamentals of Vehicle Dynamics*, Society of Automotive Engineers, Inc., 1992.
3. Genta, G., *Motor Vehicle Dynamics: Modeling and Simulation*, World Scientific Publishing, 1997.

CALIBRATION OF PROXIMITY SENSOR

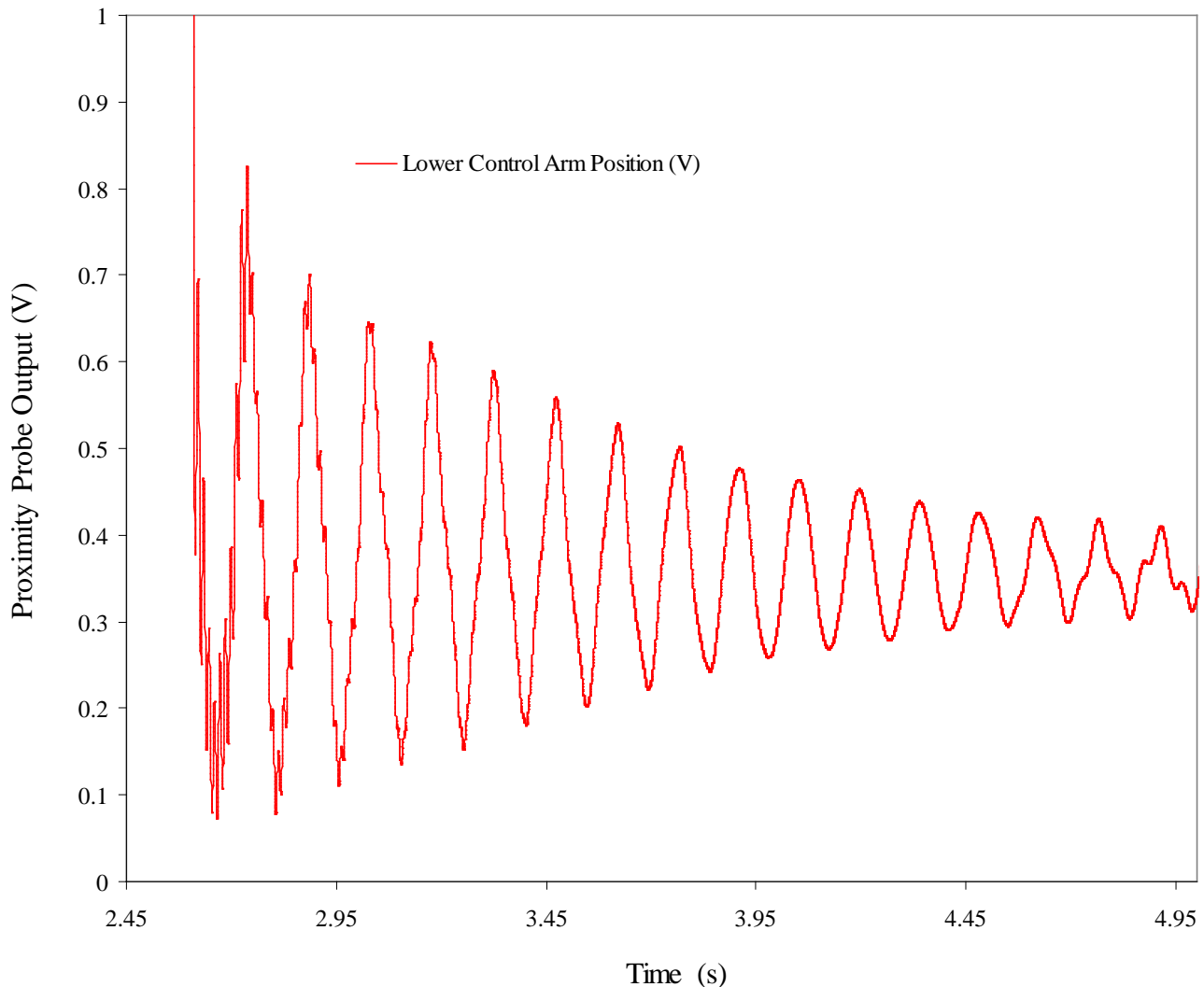
EQUIPMENT

Position sensor calibration stand includes a micrometer with a steel target. Sensor voltage is recorded using the NI-6009 DAQ card. The sensors are sensitive to the properties of the material used as a target, so the calibration material and control arm material must be consistent.

PROCEDURE

Become familiar with the use of the micrometer and proximity sensor displacement range.

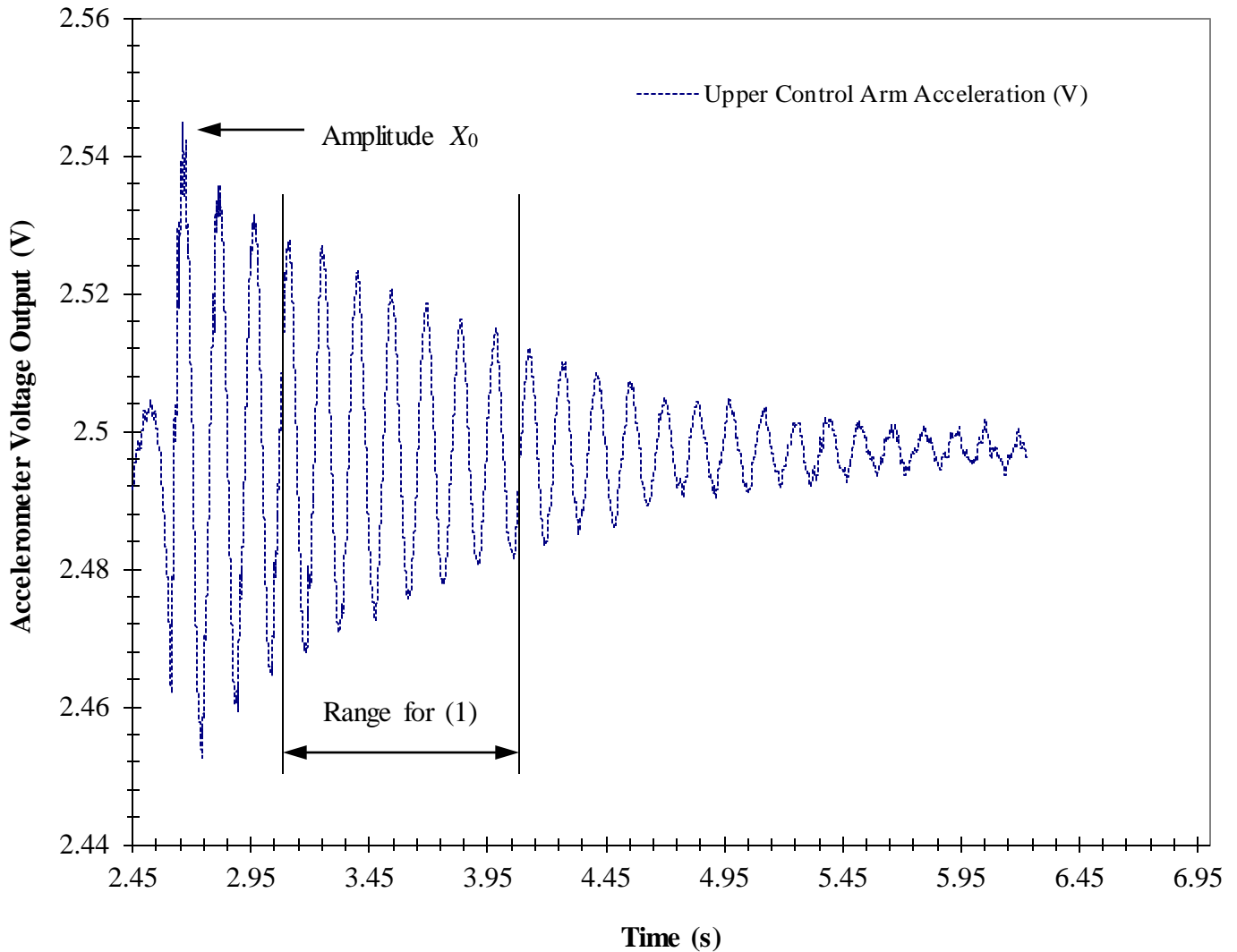
- The proximity probe will be positioned facing the bottom surface of the lower control arm below the accelerometer. It should be mounted with a slight offset from the bottom surface of the arm. When the motor is running, the sensor will produce a signal output similar to the figure below. Consult the datasheets provided on the course website for a typical output range. Your calibration should cover the range from the minimum output voltage (when the calibration target is in contact with the probe, essentially 0V) to its full maximum output (where the output voltage begins to level off).



- Mount the sensor in the calibration stand. The sensors are set up to be read as a differential voltage. The DAQ reports the voltage difference using high and low inputs on the same channel. Be sure the sensors are connected both to the power supply and to the high/low voltage inputs on the DAQ card. Consult the specification sheet on the course website for setting the power supply voltage.
- Take voltage measurements at 10 different positions to cover the full displacement range of the sensor. Be sure to record the initial micrometer reading as your “zero position”. All calibration positions will be measured relative to this initial displacement. Record the data using the VI. Record upscale/downscale cycles for a total of five data sets to check for hysteresis. Determine the full range of the sensor. Comment on any hysteresis observed in the sensor readings.
- Using a spreadsheet, calculate the average voltage for each displacement and generate a calibration equation using these values. Plot the these voltage outputs versus displacement, and obtain a linear correlation for voltage output versus displacement, using Excel’s Trendline function (in the format of $y = a + bx \pm c$).

Comment on (a) whether the data exhibit any hysteresis, (b) precision of the calibration measurements over the displacement range (repeatability), and (c) uncertainty of the linear fit (random error). Include appropriate figures in your log book.

Pre-Lab Question



(1) Using the data plotted above, choose several consecutive cycles (say ~5) in the amplitude range indicated. Divide the number of cycles by the time taken to complete them being sure to take beginning and end times from the same phase of the respective cycles. Estimate the data value with reasonable accuracy from the plot, nothing more than a ruler should be necessary. Convert the resulting frequency in Hz to radians/sec. This *damped frequency*, ω_d , approximates the *natural frequency*, ω_n , according to:

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} \approx \omega_d \text{ for small } \zeta \quad (1)$$

What is the damped frequency in rad/sec?

(2) Using the initial cycle amplitude X_o and the n -th cycle amplitude X_n for n cycles you can select in the figure above, find the damping ratio ζ_m using the relationships associated with the *logarithmic decrement*:

$$\frac{\zeta_m}{\sqrt{1-\zeta_m^2}} = \frac{1}{2\pi n} \ln \frac{X_o}{X_n} \quad \rightarrow \quad \zeta_m = \frac{1}{2\pi n} \ln \frac{X_o}{X_n} \quad (\text{for small } \zeta_m) \quad (2)$$

Compare the ζ_m value from the expression on the right in Eq. (2) with the small ζ_m assumption expression on the left and see if this value is small enough so the approximations of Eq's (1 and 2) are valid.

PROCEDURE FOR DAY 2

- a. Determine the spring constant, k (i.e. in units of N/m) for the spring, the effective mass of the upper control arm m_u and the damping coefficient c for the upper control arm. See the section “MEASURE SUSPENSION CHARACTERISTICS” that follows for this procedure.
- b. Operate the system as close as possible to the resonance state. Compared measured data with theory based on the suspension characteristics measured in parts (a). Investigate and compare the data acquired by the sensors reporting on your observations.

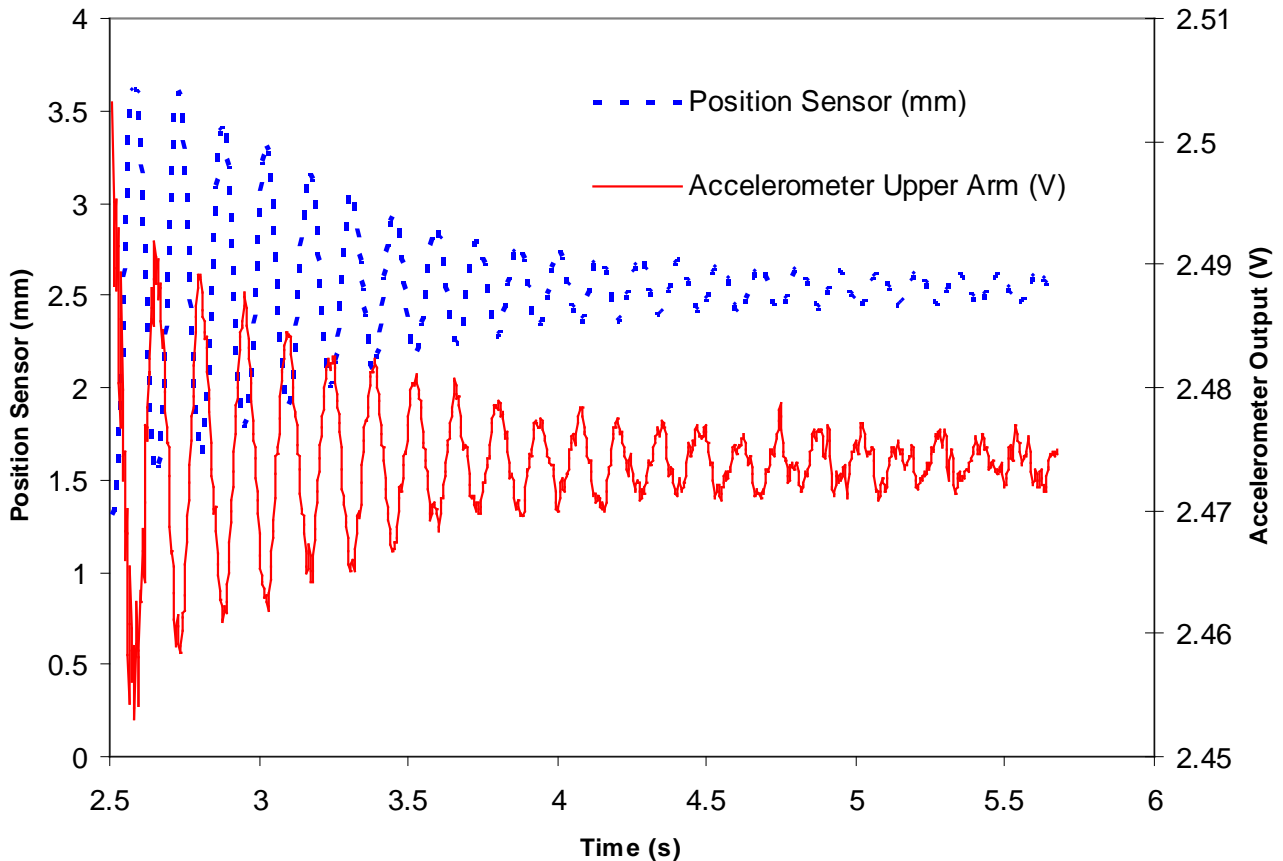
MEASURE SUSPENSION CHARACTERISTICS

This section gives a procedure for identifying the suspension parameters: the spring constant, damping coefficient and effective mass of the upper control arm. You will measure the spring constant by loading the upper control arm with masses and measuring its displacement. The method used will be to indirectly measure the control arm mass, spring, and damping parameters by making acceleration measurements of the system subject to small step inputs made by causing the upper control arm to oscillate.

Procedure:

1. Identify the four brass masses and weigh them to determine an accurate mass of each. Record these masses in your spreadsheet.
2. Determine the spring constant by stacking various combinations of the masses on the upper control arm.
 - It is recommended that you begin by placing all the masses onto the arm. Then position the sensor as close as possible to the bottom surface of the upper control arm while getting a good signal, as low as possible in your calibration range. As the masses are removed, the arm will move away from the sensor, which will give the displacement relative to each loaded case.
 - By determining the change in load divided by the change in displacement you can determine the spring constant for different cases (in kN/m).
 - Take sufficient readings to obtain at least six estimates of the spring constant. Comment on any differences among estimates. Do differences appear to be random or systematic?
 - After completely unloading the arm, fully load it again to check if the displacement has returned to the original fully-loaded starting point.
 - Estimate the spring constant and its uncertainty.
3. Place all masses on the upper control arm and lock them down with the wing nut.
4. Test your ability to operate the motor which will produce the disturbance to the system. At the start the motor should be unplugged. Before plugging it in, make sure the power is switch off, the speed control knob is fully turned counter clockwise, and that all the run switches on the lower part of the control box are

set to the run/up position. Then plug in the motor control box and turn the power on. Slowly turn the control speed until the motor begins to spin, just fast enough to overcome the disturbance wheel resistance. Stop the motor using the left-most run toggle switch. Since the motor turns clockwise, rotate and position the disturbance on the wheel to a point just past the wheel on the lower control arm. Using the run toggle switch you can operate the motor to put a single disturbance into the system, by running the motor just long enough for the “speed bump” to set the system into oscillations before toggling it off. Then you can re-position the “speed bump” for the another run.



4. Use the LabView VI to record the acceleration and position of the upper control arm. Remember that if the VI is run without changing the name of output file, the previous data will be overwritten.

Note: Depending on the position of the proximity sensor, the observed displacement by the VI may not be correct. This is because the displacement at the sensor location is smaller than the displacement at the spring. Use geometry and the calibration curve to translate the proximity sensor voltage reading to displacement at the spring.

5. Choose several (say 3 to 6) consecutive cycles of data. Use the data which appears best, position or accelerometer, and in a good amplitude range as indicated in the figure above. Since smaller amplitude responses become dominated by nonlinear friction effects, they do not reflect the relevant system dynamics. Divide the number of cycles by the time taken to complete them being sure to take beginning and end times from the same phase of the respective cycles. Convert the resulting frequency in Hz to

radians/sec. This *damped frequency*, ω_d , approximates the *natural frequency*, ω_n , according to:

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} \approx \omega_d \rightarrow \text{for small } \zeta$$

For the same dataset, calculate the frequency at least five times to determine uncertainty.

6. From your data taken in step 4, measure the initial cycle amplitude X_0 and the last cycle amplitude X_n for the n cycles and using relationships associated with the *logarithmic decrement*:

$$\frac{\zeta}{\sqrt{1 - \zeta^2}} = \frac{1}{2\pi n} \ln \frac{X_0}{X_n} \rightarrow \zeta = \frac{1}{2\pi n} \ln \frac{X_0}{X_n} \quad (\text{for small } \zeta) \quad (2)$$

find the damping ratio ζ and show that for this small value the approximations of Eq's (1, 2) are valid. For the same dataset, calculate the damping ratio at least five times. You now have values of ω_n and ζ for this case. Call it case 1.

7. Remove and reconfigure the masses to produce another load case (i.e. just remove mass D for instance). Repeat Steps 4 through 6 to obtain ω_n and ζ for this case. Call it case 2.
8. Remove and reconfigure the masses to produce another load case (i.e. just remove mass A for instance). Repeat Steps 4 through 6 to obtain ω_n and ζ for this case. Call it case 3.
9. From the three load cases, and assuming $\omega_n \approx \omega_d$, you can calculate the effective mass of the upper arm for each case using (note that the masses would change if your cases do not correspond with the three described above)

$$\begin{aligned} \frac{k}{m_{\text{upperarm}} + m_{A+B+C+D}} &= \omega_{n,\text{case1}}^2 \\ \frac{k}{m_{\text{upperarm}} + m_{A+B+C}} &= \omega_{n,\text{case2}}^2 \\ \frac{k}{m_{\text{upperarm}} + m_{B+C}} &= \omega_{n,\text{case3}}^2 \end{aligned} \quad (3)$$

or

$$\begin{aligned} m_{\text{upperarm}} &= \frac{k}{\omega_{n,\text{case1}}^2} - m_{A+B+C+D} \\ m_{\text{upperarm}} &= \frac{k}{\omega_{n,\text{case2}}^2} - m_{A+B+C} \\ m_{\text{upperarm}} &= \frac{k}{\omega_{n,\text{case3}}^2} - m_{B+C} \end{aligned}$$

10. From the three load cases, and using the measured ω_d and ζ , you can calculate the effective mass of the upper arm for each case using

$$\begin{aligned} \left(\frac{k}{m_{\text{upperarm}} + m_{A+B+C+D}} \right) (1 - \zeta_{\text{case1}}^2) &= \omega_{d,\text{case1}}^2 \\ \left(\frac{k}{m_{\text{upperarm}} + m_{A+B+C}} \right) (1 - \zeta_{\text{case2}}^2) &= \omega_{d,\text{case2}}^2 \\ \left(\frac{k}{m_{\text{upperarm}} + m_{B+C}} \right) (1 - \zeta_{\text{case3}}^2) &= \omega_{d,\text{case3}}^2 \end{aligned} \quad (4)$$

or

$$\begin{aligned} m_{\text{upperarm}} &= \frac{k(1 - \zeta_{\text{case1}}^2)}{\omega_{n,\text{case1}}^2} - m_{A+B+C+D} \\ m_{\text{upperarm}} &= \frac{k(1 - \zeta_{\text{case2}}^2)}{\omega_{n,\text{case2}}^2} - m_{A+B+C} \\ m_{\text{upperarm}} &= \frac{k(1 - \zeta_{\text{case3}}^2)}{\omega_{n,\text{case3}}^2} - m_{B+C} \end{aligned}$$

Note that this does not use the assumption that $\omega_n \approx \omega_d$. Again, note that the masses would change if your cases do not correspond with the three described above.

11. For each of the three load cases, and using the measured ζ and the appropriate masses and an average mass from step 10 for the upper arm, you can calculate the coefficient damping c from

$$\begin{aligned} c &= 2 \zeta_{\text{case1}} \sqrt{k(m_{A+B+C+D} + m_{\text{upperarm}})} \\ c &= 2 \zeta_{\text{case2}} \sqrt{k(m_{A+B+C} + m_{\text{upperarm}})} \\ c &= 2 \zeta_{\text{case3}} \sqrt{k(m_{B+C} + m_{\text{upperarm}})} \end{aligned} \quad (5)$$

where the masses would change if your cases do not correspond with the three described above.

Now all dynamic parameters have been identified! Values for m_{upperarm} and c should be constant within the uncertainties of the system. Check for this, and note if any differences appear outside the range of uncertainty or appear to be due to systematic effects (for example do the values appear to be dependent on the applied load or natural frequency?).

12. Operate the system and motor at the lowest speed possible using the masses from one of the load cases you used in the system characteristic measurements. This should be approximately 77 rpm. You can measure the rpm speed using the optical speed sensor by aiming it at the strip of aluminum tape on the rim of the wheel. With the motor set to the same speed (do not move the speed control knob, just use the leftmost "Run"/"Coast to Stop" toggle switch), measure the position amplitude using the

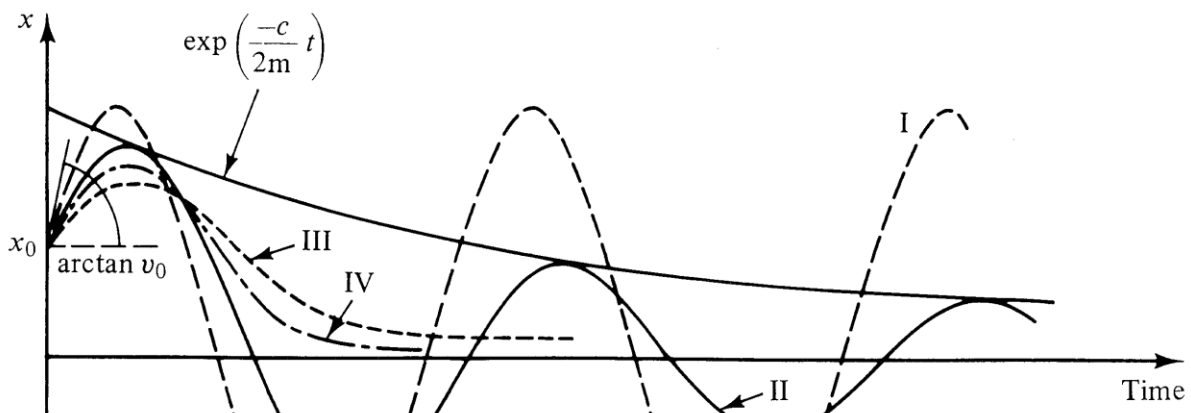
position sensor on the upper and then lower control arms. If there is only one proximity sensor available, you will have to store two sets of data.

13. Operate the motor at the higher speed where the upper arm oscillations are at a local maximum. If you increase the speed beyond this point, the amplitude on the oscillations decrease. Say for example, the damped natural frequency for your mass case is 7.14 Hz. At a rotational speed of about 86 rpm, 1.43 Hz, this corresponds to 7.14/5 Hz the 5th sub harmonic. Repeat the data acquisition from step 12 for the position amplitudes of the upper and lower arms.
14. Repeat step 13, but at a point where the oscillations are at a local minimum.
15. Outside of lab, make your own frequency response plot analogous to Figure 3.16 in your text for the Magnitude ratio. The magnitude ratio is the output/input, or in this case the upper arm displacement amplitude/lower arm displacement amplitude. The plot will differ from Figure 3.16 in your textbook because it is a periodic impulse excitation not the sinusoidal input. Your plot will have the experimental data points corresponding to the frequency ratio (ω/ω_n) used in steps 12, 13 and 14 and the magnitude ratio found in the data.

Analysis

In your log book and/or report, discuss and present the calculated parameters mentioned in this procedure for the mass/spring/damper system.

- a. Record and transfer all data to computer for analysis using the LabView VI. Use three mass loadings. Store all data for analysis.
- b. Discuss and present your observations of the measurements, including displacement and acceleration. Make an example plot the acceleration and position data so they can be directly compared.
- c. Based on an appropriate physical model, discuss the "apparent" damping of the system, e.g., Coulomb versus viscous damping. See pages 53-58 of the lecture notes on the course website for the theoretical development of this dynamic system. Frictional damping is shown using curve II in the plot below; for this damping the cyclic peaks decay according to the function $\exp(-ct/2m)$, as is also shown below. Coulomb damping is a constant frictional damping where the cyclic peaks decay linearly.



- d. Compare the measured effective upper arm mass and the damping ratio. They should be constant within the uncertainty of the experiment. Discuss them. Do they appear to be dependent on some experimental variable? How might the data be analyzed differently?
- e. Discuss uncertainty of measurements.